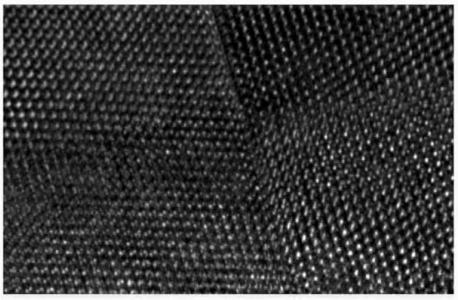
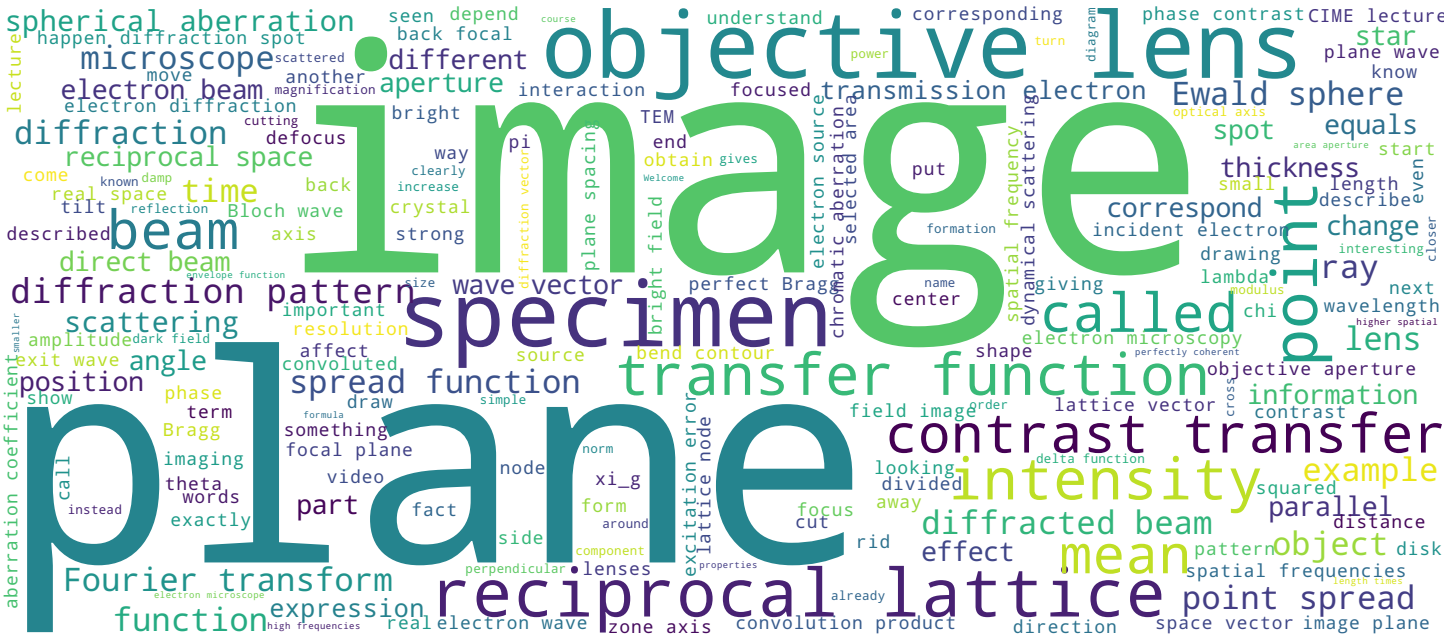


Phase contrast: Contrast Transfer Function

Transmission Electron Microscopy

Prof. C. Hébert & Dr. D. Alexander



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Video

EPFL

Introduction

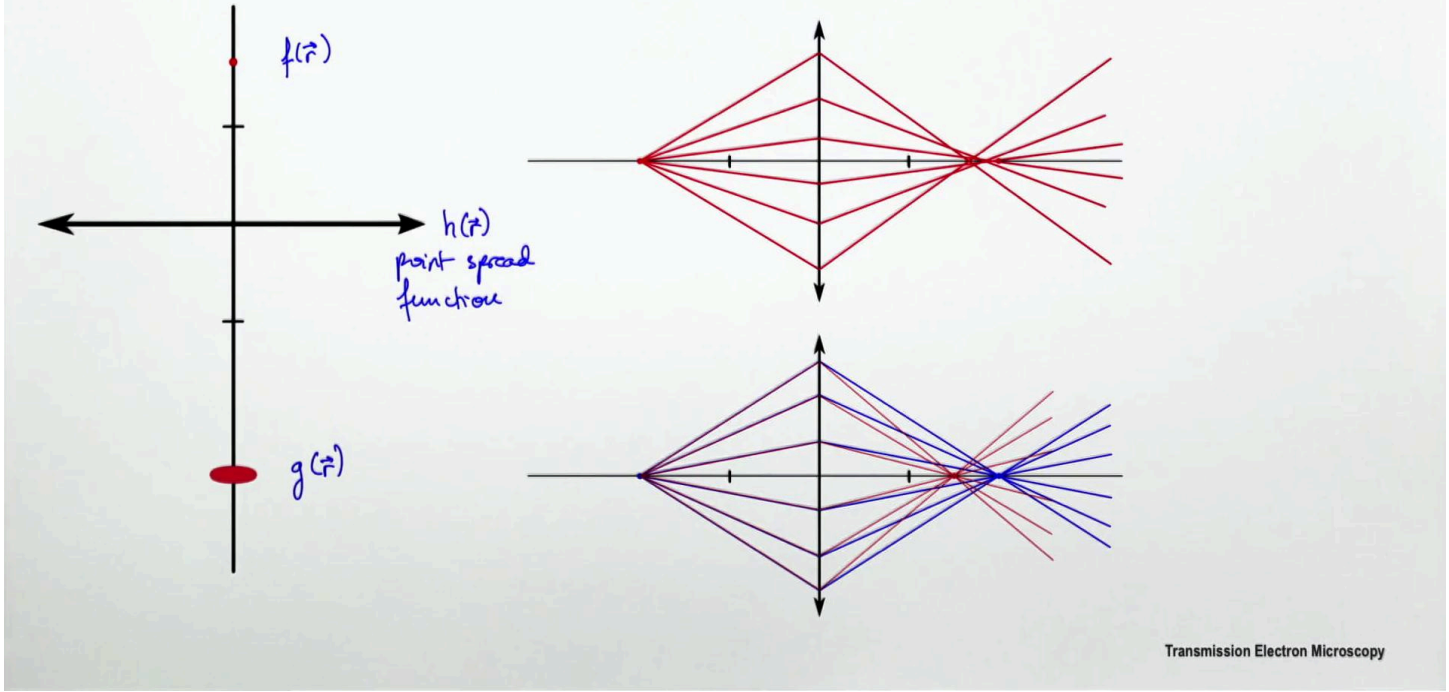
Welcome to CIME's lecture on Transmission Electron Microscopy for material science. In this video will we see how to describe the objective lens with the concept of point spread function and contrast-transfer function.

Notes

Summary

0m 04s

Point spread function



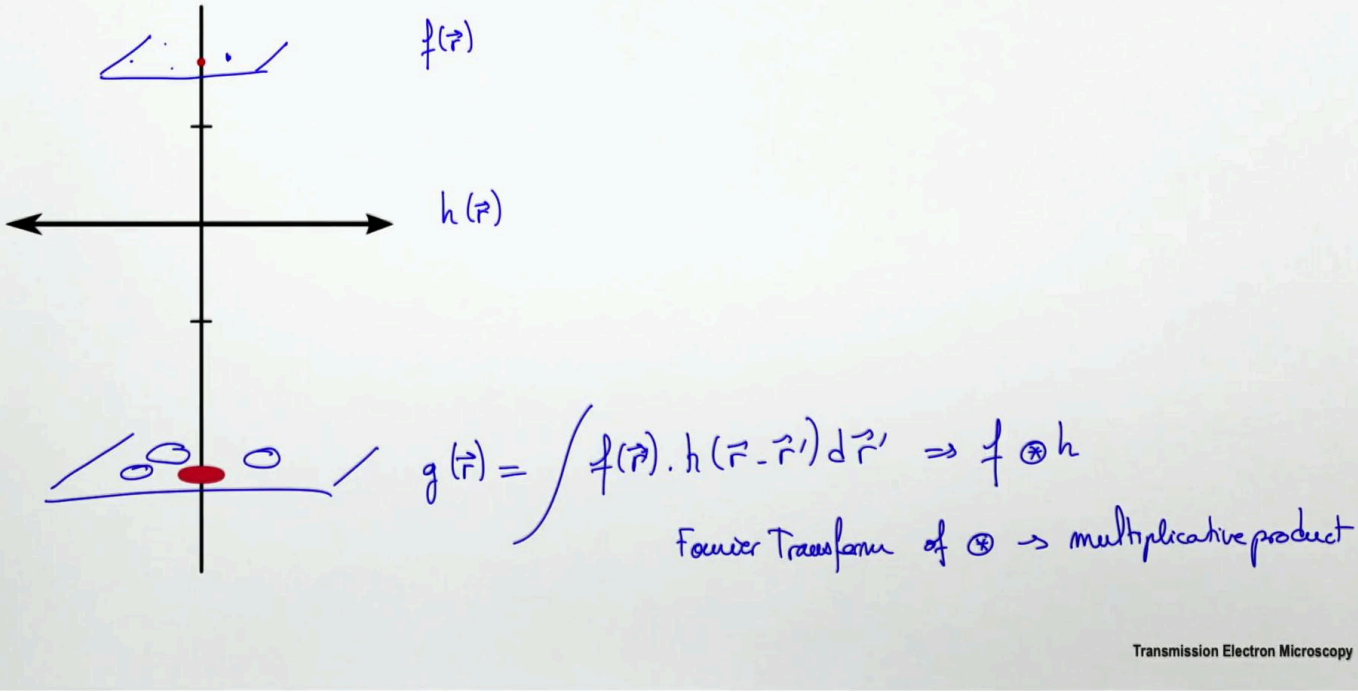
So, let's start with the point spread function. As you can guess from the name, it will tell us how the point is then spread, which means that we first start with our objective lens and want it to make the image of a point. The image of a point by the objective lens should normally, if we have a perfect lens, be a point like this. But you already have seen in the part about lens aberrations that we have, for example, the spherical aberration. With the spherical aberration, the image of a point is spread into a disk of least confusion. We also had the case of chromatic aberration where rays of different energies are not focused at the same position. Again this spreads the information. And finally we may change a little bit the focus of the lens, which means that we move the position where the image forms and at the previous position of the image, we will have a disk. Finally, the image of a point will turn into a disk. Let's put some names on all of this. For example, our object can be described by the function f(r), with r the position in the plane. The image of our object will be called g(r) in real space. If our object was a perfect point, the image "g" corresponds exactly to what we call the point spread function h(r), which is a characteristic of the objective lens. But it is not very interesting to image point objects.

Notes

Summary

0m 22s

Point spread function



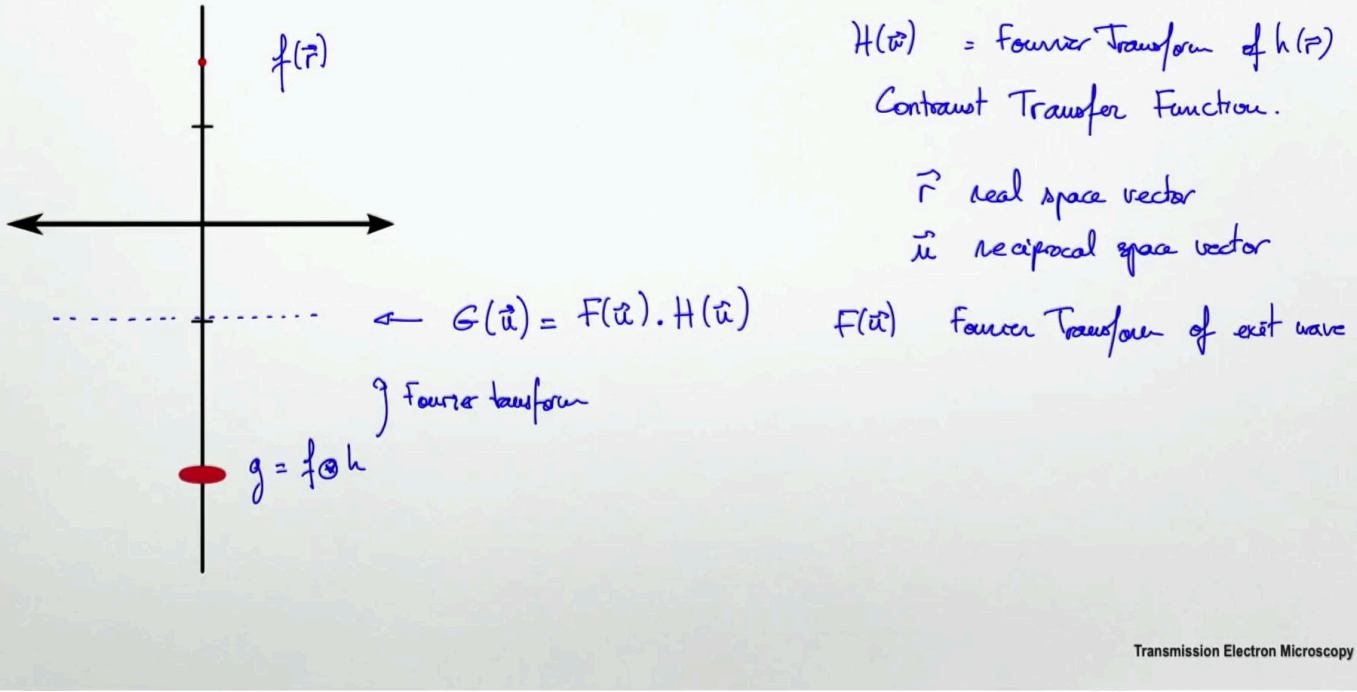
What is going on for an extended object? Instead of that point, I now have a plane and I have to consider different points of this plane. The image will be situated in the image plane and actually, each of the points of my object will be spread by the point spread function in the corresponding image plane. At the end, I have all this information mixed up. And how is it mixed up? Well, if I have my object f(r), my point spread function h(r) describing the objective lens, at the end, in the image plane, I will get my image called g(r), which will be the integral over the whole image of all this mixed information, which can be described by: f(r') times h(r- r')dr'. This is a convolution product which we can write: f convoluted with h. So the image is the convolution product of the object with the point spread function. It would be easier if we could get rid of this convolution product. You have certainly seen in Fourier analysis that the Fourier transform of a convolution product becomes a normal multiplicative product. So we will use this to get rid of this convolution.

Notes

Summary

2m 30s

Contrast Transfer Function

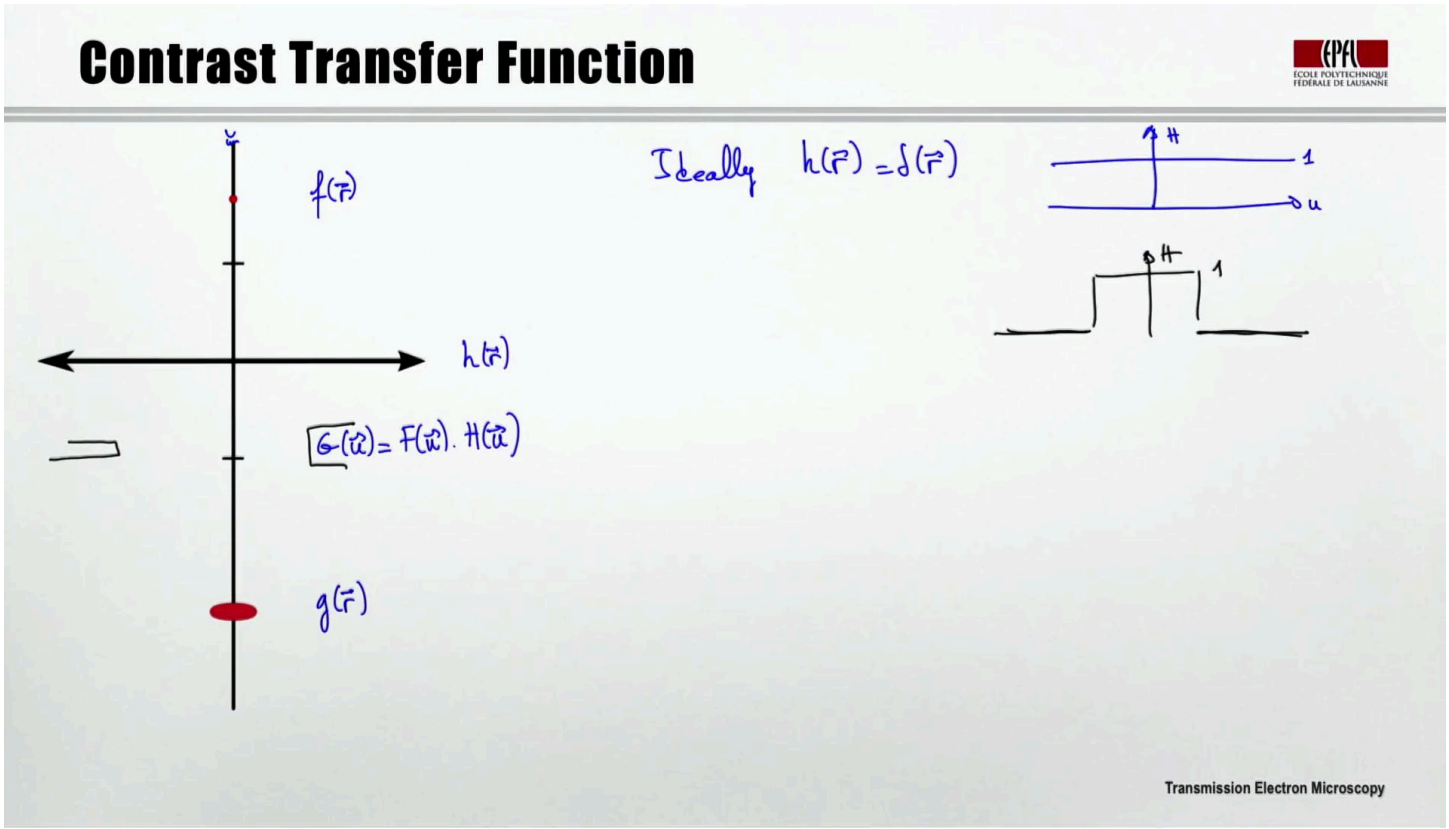


Again we have f(r), our object, g which is equal to f convoluted with h, our image. And now to get rid of this convolution product, I will take the Fourier transform and that would give me a quantity called G(u) = F(u) times H(u). H(u) is the Fourier transform of h(r). It is called the contrast transfer function, but it is also connected to reality. If we have a real space vector r, u will correspond to spatial frequencies. It is what we find in the reciprocal space: r is a real space vector, u is a reciprocal space vector. Reciprocal space is found in the back focal plane of the objective lens. So, what happens there can be understood by looking at the back focal plane of the objective lens. F(u) is the Fourier transform of the exit wave. It carries exactly the same information as the exit wave, that is the information we want to access from the specimen. So now, we will have a closer look at the contrast transfer function.

Notes

Summary

4m 28s

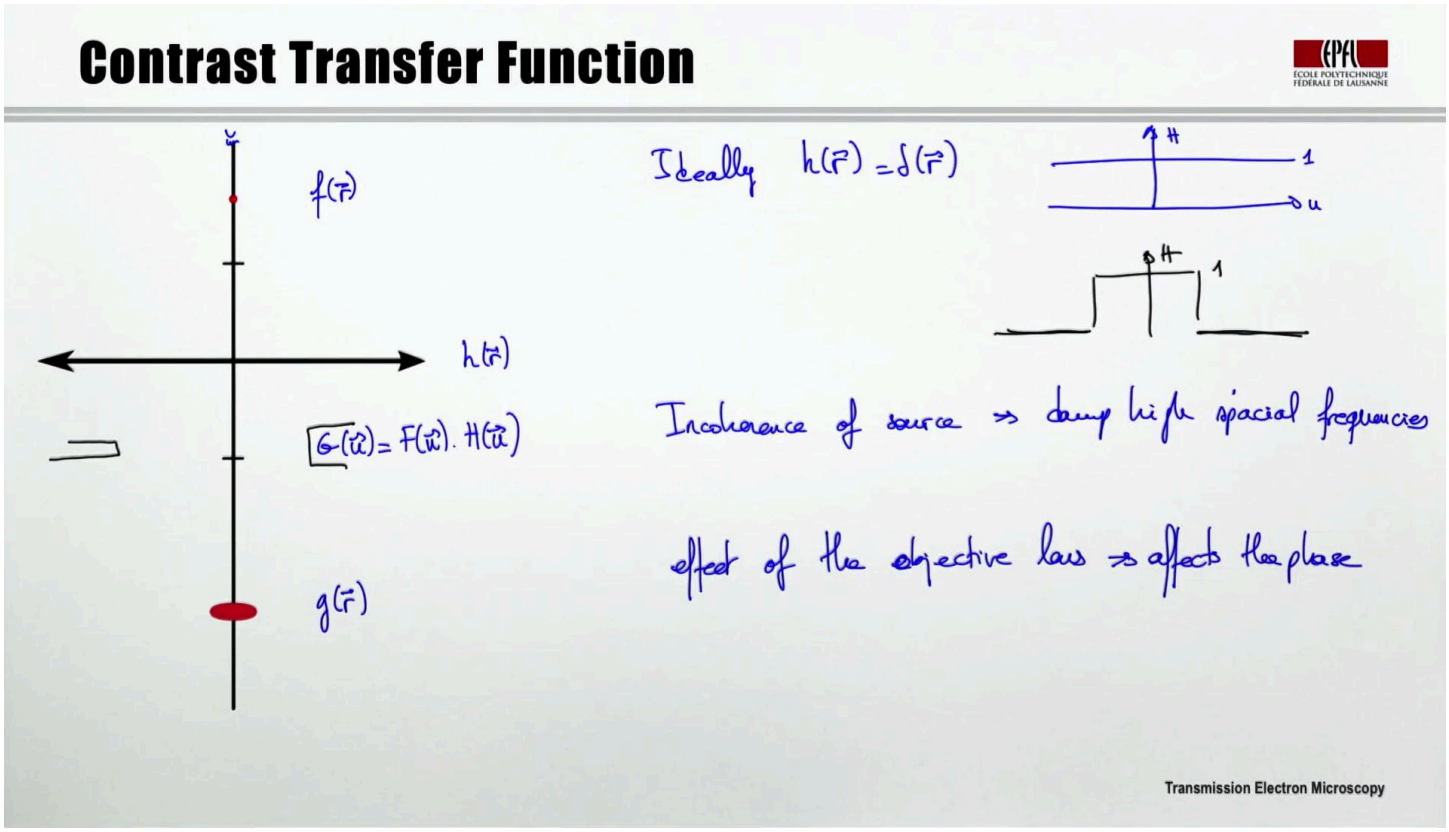


Back to real space, ideally, we would have to have the image of a point, not being a disk but a point, which means that we would like to have the point spread function not spreading anything, so, being a delta function. The Fourier transform of a delta function is unity. Ideally, the contrast transfer function should be one and constant, which means that I should be able to transmit equally each spatial frequency. But it is impossible in a transmission microscope. First of all, I'm always using an objective aperture, and even if not I will have some cut off by the wall of the objective lens pole pieces, for example. So, I'm always cutting the rays at some angle which means that I will always cut somewhere the spatial frequencies. My contrast transfer function will be cut and look like this. And, at the higher spatial frequencies everything is gone because of the aperture. Secondly, my electron source is not perfect either. Electrons will, for example, be emitted from several places out of the electron source. As a consequence, electron waves coming from different places of the source will not be perfectly coherent with each other.

Notes

Summary

6m 16s

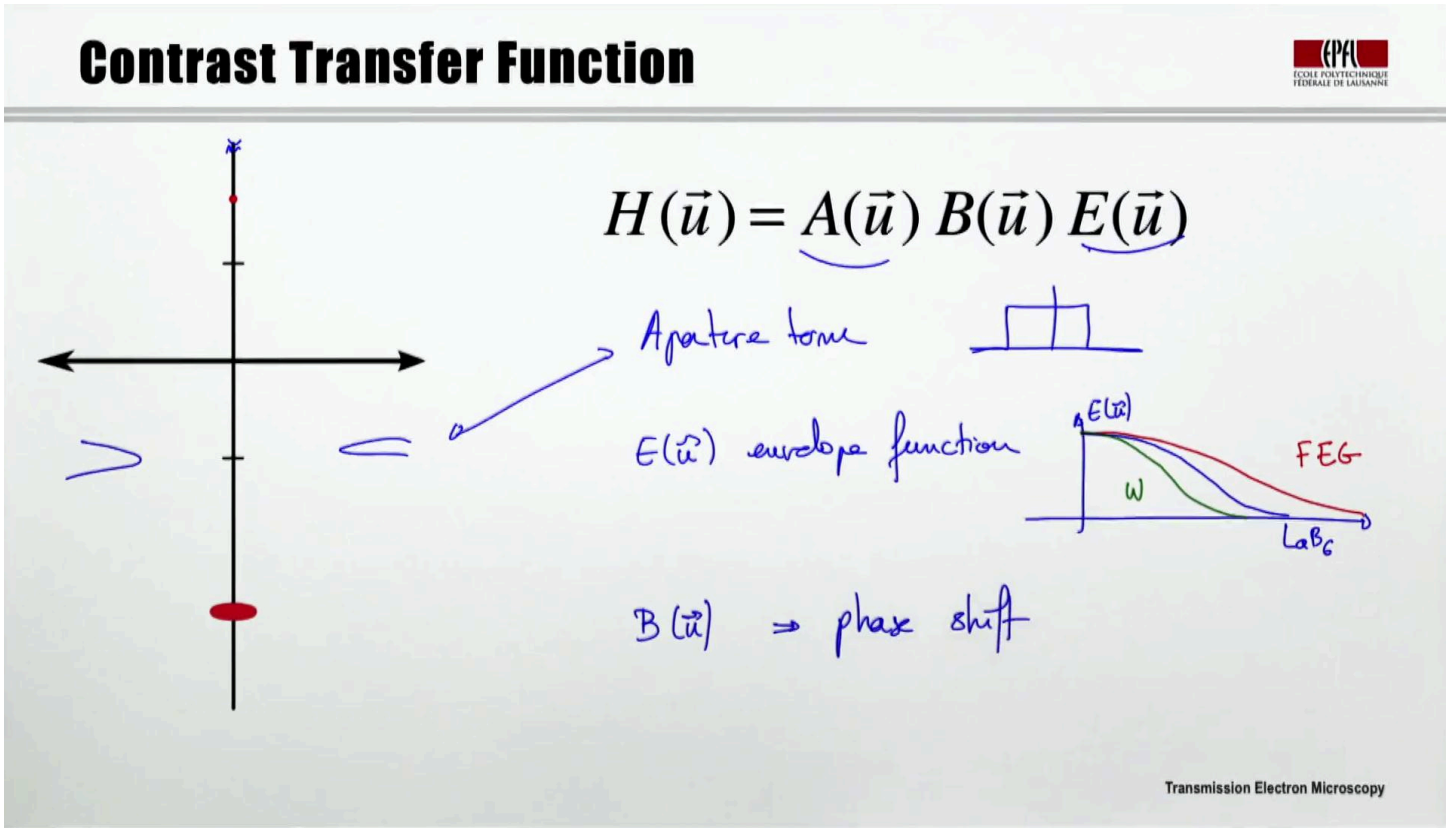


If I want to resolve very fine details, which means have a good transfer function at very high frequency, I will have a problem. That is called the incoherence of the source and this would damp the higher frequency. And finally, we also have to take into account the objective lens. The objective lens will not affect the amplitude of the electron wave, but it will affect the phase, and it would affect the phase in a very complicated manner that depends on the reciprocal space vector, or on the angle at which the electrons are scattered. Finally, I can model my contrast transfer function with those three components in the following way.

Notes

Summary

7m 56s



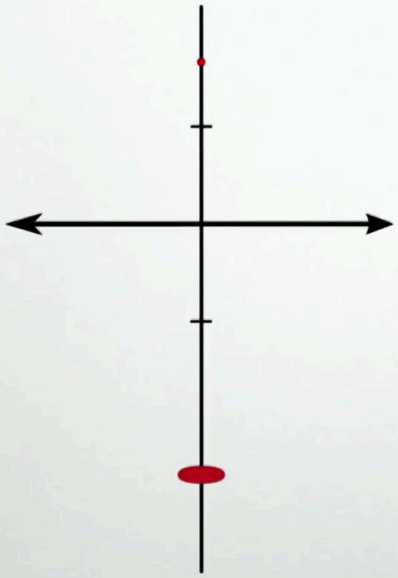
I use three terms called A, B, and E. The first one is the aperture term. It corresponds to the objective aperture in the back focal plane of the objective lens and it will be a square top hat function. it is a hard cutoff at high frequencies. The second term is called the envelope function. The envelope function describes the damping at higher spatial frequency due to the fact that the source is not perfectly coherent. For example, if we look at different kind of electron sources that we have classically in electron microscopy, we can compare, for example, the field emission gun which has a very high coherency, so it will only damp at relatively high frequencies with lanthanum hexaboride gun which has lower coherency and will end up with a stronger damping at high spatial frequency, or even worse tungsten filament that will damp even more. Finally, I have to consider now B(u). This is the term describing the effect of the objective lens and this one only applies a phase shift, which means that I can simplify its expression and write it as exponential i chi(u).

Notes

Summary

8m 58s

Contrast Transfer Function



$$H(\vec{u}) = A(\vec{u})\, B(\vec{u})\, E(\vec{u})$$

$$B(\vec{u}) = e^{i\chi(\vec{u})}$$

- If we only have defocus and spherical aberration:

$$\chi(\vec{u}) = \pi \Delta f \lambda u^2 + \frac{1}{2}\pi C_s \lambda^3 u^4$$

λ : wavelength

Transmission Electron Microscopy

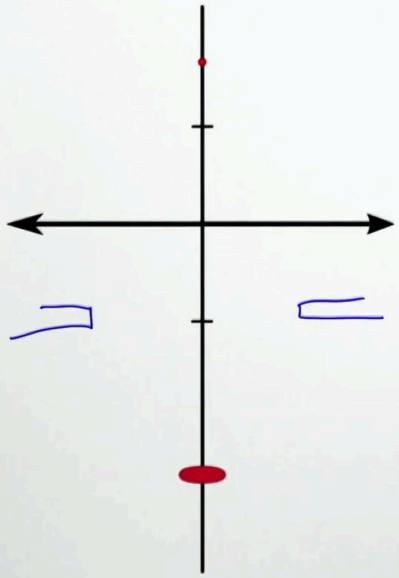
Chi is a real function that depends on the properties of the lens. For example, chi would depend on the spherical aberration, chromatic aberration, astigmatism, etc. If we only have defocus of the lens and spherical aberration, which are two aberrations with cylindrical symmetry, so I don't have to take care of the vector nature of u and the direction. All you need to know is it is norm. In that case, the expression of chi is given by this formula which contains the defocus applied to the objective lens, the spherical aberration coefficient C_s, the wavelength of the electrons, lambda, and the norm of the reciprocal space vector, u. The dimension analysis tells us that everything is okay. This is a length. This is the length times a reciprocal length squared that give something which is unitless and there I have length times length to the power of three compensated by this reciprocal unit vector to the power of four. It is interesting to note that this is growing extremely fast with u. That explains to you the importance of the spherical aberration coefficient and the fact that if you have a microscope with strong C_s, very soon your resolution will be limited by the C_s or you will have to cut u.

Notes

Summary

10m 47s

Contrast Transfer Function



$$H(\vec{u}) = A(\vec{u})\, B(\vec{u})\, E(\vec{u})$$

$$B(\vec{u}) = e^{i\chi(\vec{u})}$$

- If we only have defocus and spherical aberration:

$$\chi(\vec{u}) = \pi \Delta f \lambda u^2 + \frac{1}{2}\pi C_s \lambda^3 u^4$$

λ : wavelength

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But, cutting u will not help you any further. Cutting the high values of u means introducing an aperture and what you gain on one side by reducing this term, you lose it on the other side by having the aperture function that would also kill the resolution of the microscope.

Notes

Summary

12m 49s

With this you have had the notion of the contrast transfer function which is a very important quantity to describe how well the different spatial frequencies are transferred by the optical system. In the next video, we will introduce the interaction between the electron plane wave and the specimen.

Notes

Summary

13m 19s